PULSED-VOLTAGE STUDIES OF AN ION-EXCHANGE PUMP WITH AN AXISYMMETRIC COLLECTOR

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Elsewhere [1] we studied an ion-convection pump (ICP), a system consisting of a needle and a coaxial cylinder with a cylindrical channel, and constructed a model of the energy conversion process for calculating the pressure developed by the pump, depending on the geometric parameters of the flow section and the electrical characteristics of the supply voltage. The method used there, based on calculation of the average potential U(x, t), does not work for channels with a variable cross-sectional area S(x) of the flow section.

In this paper we propose a modified analytic treatment, which is based on calculation of the average electric field strength E(x, t) and is intended for problems of optimization of the shape of the ICP collector electrode. The results of [1] follow from the formulas derived below for a particular case (for $S(x) \equiv \text{const}$).

1. Electric Field Strength in an Axisymmetric ICP. We consider an axisymmetric channel K of length L with a conducting lateral surface δK and a metal needle on the axis of the channel (Fig. 1). The channel is filled with a viscous incompressible liquid; a voltage U(t) pulsating with frequency ω maintained between the needle and the surface δK injects charges into the liquid near the needle and causes an electrohydrodynamic (EHD) flow. We find the average electric field strength during steady-state operation of the ICP, assuming laminar motion of the liquid.

We use a nonstationary hydraulic approximation of the system of EHD equations for the ICP stage (see, e.g., [1-4]),

$$\rho\frac{\partial v}{\partial t}+\rho v\frac{\partial v}{\partial x}=\frac{\varepsilon\varepsilon_0}{S(x)}\frac{\partial}{\partial x}\left(F_i\frac{E^2}{2}\right)-\frac{\partial p}{\partial x}+\Lambda; \quad (1.1)$$

$$\frac{\partial(F_iq)}{\partial t}+\frac{\partial(F_ij)}{\partial x}=0; \quad (1.2)$$

$$j=q(v+bE); \quad (1.3)$$

$$\frac{\partial(F_iE)}{\partial x}=\frac{F_iq}{\varepsilon\varepsilon_0}; \quad (1.4)$$

$$F_iE=-\frac{\partial(F_iU)}{\partial x}; \quad (1.5)$$

$$\frac{\partial(S(x)v)}{\partial t}+v\frac{\partial(S(x)v)}{\partial x}=0 \quad (1.6)$$

for the boundary condition $\partial E/\partial t|_{x=0}=0$. Here S(x) is the cross-sectional area of the pump channel; v = v(x, t) is the cross-sectional average velocity of the liquid; $F_i = F_i(x, t)$ is the cross-sectional area of the charge-exchange zone; ρ and p are the average density and pressure of the neutral component over the cross section F_i; E, U, q, and j are the average electric field strength, potential, charge density, and conduction current density over F_i; ε_0 and ε are the electric and dielectric constants of the liquid; Λ is the hydraulic approximation of the viscosity term in the Navier–Stokes equation; and b is the coefficient of ion mobility.

The boundary condition $\partial E/\partial t|_{x=0}=0$, being the main condition in the study of corona discharges in gases [5] and the analogous process in dielectric liquids [1, 3, 4], characterizes the effect of the intrinsic electric field of charge exchange on the electric field of the electrodes.

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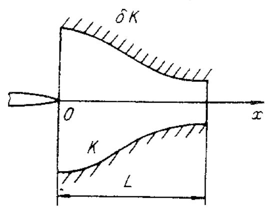

Fig. 1

We assume that the inertial forces of the liquid smooth out its velocity pulsations due to the periodicity of the supply voltage and, hence, $\partial(S(x)v)/\partial t$ is negligible. Equation (1.6), therefore implies the equation

$$v(x) = v(x,\ t) = v_0S_0/S(x) \quad (v_0 = v(0),\ S_0 = S(0)). \tag{1.7}$$

The pulsation of the supply voltage causes the ions to move along the transfer zone in batches [6], whose velocity is of the order of v(x) [4]. Formalizing this in terms of the charge density q(x, t) or in terms of the electric relaxation frequency

$$\beta(x,\ t) = bq/\varepsilon\varepsilon_0, \tag{1.8}$$

we find that the function β is periodic in both arguments with periods

$$T = 2\pi\omega^{-1},\ X = \text{const}\ T. \tag{1.9}$$

From Eqs. (1.2) and (1.4) we obtain

$$\frac{\partial}{\partial x}\left(\varepsilon\varepsilon_0\frac{\partial(F_iE)}{\partial t} + F_ij\right) = 0. \tag{1.10}$$

Suppose that J(x, t) is the average density of the total current over the cross section $F_i(x)$. Then the expression in parentheses in formula (1.10) is equal to F_iJ and, therefore, we have

$$\varepsilon\varepsilon_0\frac{\partial(F_iE)}{\partial t} + F_ij = F_i(0,t)J(0,t). \tag{1.11}$$

Introducing the notation $\hat{E} = F_iE$, $\hat{q} = F_iq$, and $\hat{j} = F_ij$ and using (1.3), (1.4), (1.8), and (1.11), we obtain

$$\frac{\partial\widehat{E}}{\partial t} + v\frac{\partial\widehat{E}}{\partial x} + \beta(x,t)\widehat{E} = \frac{1}{\varepsilon\varepsilon_0}F_0J_0(t) \tag{1.12}$$
$$(F_0 = F_i(0,\ t),\ J_0(t) = J(0,\ t)).$$

By virtue of the periodicity of β with periods (1.9) and the principle of averaging for hyperbolic equations (see, e.g., [7]) the solutions (1.12) converge to solutions of the averaged equation at a sufficiently high frequency $\omega \to \infty$:

$$\frac{\partial\widehat{E}}{\partial t} + v\frac{\partial\widehat{E}}{\partial x} + \overline{\beta}\widehat{E} = \frac{1}{\varepsilon\varepsilon_0}F_0J_0(t)$$
$$\left(\overline{\beta} = \frac{1}{XT}\int_0^X\int_0^T\beta(x,\ t)\,dx\,dt\right). \tag{1.13}$$

We find the solution of (1.13) with the boundary condition $E(0,\ t) \equiv E_0$, using the standard method of characteristics. The characteristic system of ordinary differential equations for (1.13) has the form

$$dt = \frac{dx}{v(x)} = \frac{du}{\frac{1}{\varepsilon\varepsilon_0}F_0J_0 - \overline{\beta}u}. \tag{1.14}$$

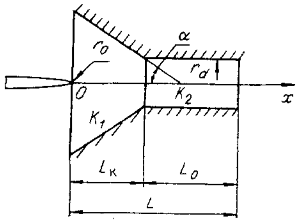

Fig. 2

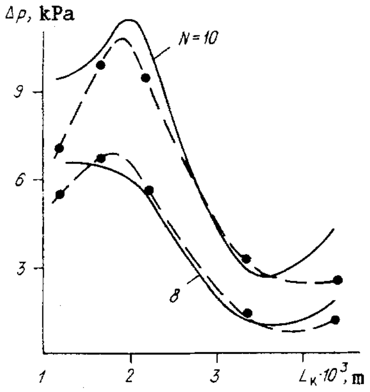

Fig. 3

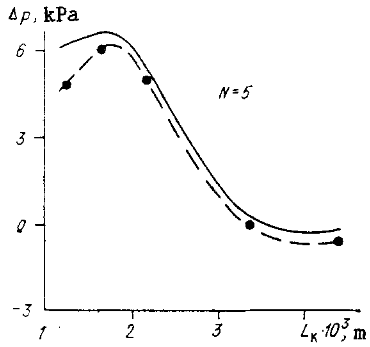

Fig. 4

We define two independent integrals of a given system. The first integral of (1.14) can be written as

$$t - \int_0^x \frac{dy}{v(y)} = C_1. \tag{1.15}$$

We note that $\tau(x_0) = \int_0^{x_0} (dy/v(y))$ is the time during which the liquid traverses the part of the channel from $x = 0$ to $x = x_0$. Taking (1.7) into account, we write this as

$$\tau(x) = \frac{1}{v_0 S_0} \int_0^x S(y)\, dy. \tag{1.16}$$

The second integral is the general solution of the linear equation

$$u'(t) + \bar{\beta} u(t) = \frac{1}{\varepsilon\varepsilon_0} F_0 J_0(t)$$

and is found by the method of variations:

$$u \exp(\bar{\beta} t) - \frac{1}{\varepsilon\varepsilon_0} \int_0^t F_0 J_0(p) \exp(\bar{\beta} p)\, dp = C_2.$$

Since u does not appear explicitly in the first integral, the general solution of (1.13) satisfies the functional equation

$$\widehat{E} \exp(\bar{\beta} t) - \frac{1}{\varepsilon\varepsilon_0} \int_0^t F_0 J_0(p) \exp(\bar{\beta} p)\, dp = f(t - \tau(x)),$$

which contains the derivative of the smooth function f. Taking the boundary condition into account, we obtain

$$f(t)=\widehat{E}(0,t)\exp(\overline{\beta}t)-\frac{1}{\varepsilon\varepsilon_0}\int_0^t F_0J_0(p)\exp(\overline{\beta}p)\,dp=F_0E_0\exp(\overline{\beta}t)-\frac{1}{\varepsilon\varepsilon_0}\int_0^t F_0J_0(p)\exp(\overline{\beta}p)\,dp.$$

The equation

$$F_iE=\widehat{E}(x,t)=F_0E_0\exp(-\overline{\beta}\tau(x))+\frac{\exp(-\overline{\beta}t)}{\varepsilon\varepsilon_0}\int_{t-\tau(x)}^{t}F_0J_0(p)\exp(\overline{\beta}p)\,dp, \tag{1.17}$$

which is the asymptotic (relative to $\omega\to\infty$) solution of Eq. (1.12) for the average field strength in an axisymmetric ICP.

2. Calculation of Δp_{av}. Comparison with Experiment. The first aspect of this study (as well as [1]) is the problem of explaining the experimental data [6] on the abrupt increase in the pressure drop Δp at certain values of the geometric parameters of the pump stage. The formula for averaging the field strength makes it possible to solve this problem, substituting (1.17) into Eq. (1.1).

At the same time, the present lack of a single generally accepted theory of charge formation in a dielectric liquid and, therefore, the equations of the current-voltage characteristic for the ICP also required use of experimental data to calculate Δp. Since the internal processes in the ICP stage are complex and there is no additional information about the structure of the viscous term of Eq. (1.1), we use the simplest hydraulic approximation.

Integrating Eq. (1.1) with allowance for (1.7) and (1.17), we obtain

$$\Delta p(t)=\frac{\varepsilon\varepsilon_0}{2}\int_0^L\frac{1}{S(x)}\frac{d}{dx}\left\{\frac{1}{F_i}\left[F_0E_0\exp(-\overline{\beta}\tau(x))-\right.\right.$$
$$\left.\left.-\frac{\exp(-\overline{\beta}t)}{\varepsilon\varepsilon_0}\int_0^{t-\tau(x)}F_0J_0(p)\exp(\overline{\beta}p)\,dp+\frac{\exp(-\overline{\beta}t)}{\varepsilon\varepsilon_0}\int_0^t F_0J_0(p)\exp(\overline{\beta}p)\,dp\right]^2\right\}dx+ \tag{2.1}$$
$$+\rho v_0^2S_0^2\int_0^L\frac{1}{S^3(x)}\frac{dS}{dx}\,dx-\xi\frac{\rho v_*^2}{2}$$

(ξ is the hydraulic loss coefficient and v_* is the characteristic velocity of the working medium).

When the current density $J_0(t)$ at the end of the needle is known the optimization of the shape of the flow section of the pump channel (shape of the collector electrode) reduces to a study of the maximum of the functional (2.1) averaged over t. Suppose that the channel K has the form K_1+K_2 (K_1 is a cone of angle α and length L_K and K_2 is a cylinder of radius r_d and length L_0). In this case $L=L_K+L_0$ (Fig. 2).

We introduce the simplifying assumptions: 1) the current $i_0(t)$ on the needle is approximated well by the fundamental harmonic

$$i_0(t)\simeq I_0\cos\omega t;$$

2) the values of F_i are determined by

$$F_i=\begin{cases}\pi r_0^2, & x=0,\\ S(x), & 0<x\leqslant L.\end{cases} \tag{2.2}$$

Taking the assumptions and formulas (1.7) and (1.16) into account, we obtain

$$\Delta p_{av}=\Delta p=\frac{1}{T}\int_0^T\Delta p(t)\,dt=\frac{\varepsilon\varepsilon_0}{2}(A-B)-D-\xi\frac{\rho v_d^2}{2}, \tag{2.3}$$

where

$$A=\left(\frac{r_0}{r_d}\right)^4\left[E_0^2\exp\left(-2\overline{\beta}\tau(L)\right)+\frac{M(L)}{2}\left(\frac{I_0}{\varepsilon\varepsilon_0\pi r_0^2}\right)^2\right]-E_0^2;$$

$$B=2r_0^4\,\mathrm{tg}\,\alpha\int\limits_0^{L_\text{к}}\frac{E_0^2\exp\left(-2\overline{\beta}\tau(x)\right)+\frac{M(x)}{2}\left(\frac{I_0}{\varepsilon\varepsilon_0\pi r_0^2}\right)^2}{(r_d+(L_\text{к}-x)\,\mathrm{tg}\,\alpha)^5}\,dx;$$

$$D=\frac{\rho v_d^2}{2}\left[1-1\Big/\left(1+\frac{L_\text{к}}{r_d}\,\mathrm{tg}\,\alpha\right)^4\right];$$

$$M(x)=\frac{[\exp(-\overline{\beta}\tau(x))-\cos\omega\tau(x)]^2+\sin^2\omega\tau(x)}{\overline{\beta}^2+\omega^2};$$

$$\tau(x)=\frac{r_d\,\mathrm{ctg}\,\alpha\,(1+L_\text{к}\mathrm{tg}\,\alpha/r_d)^3}{3v_d}\left[1-\left(1-\frac{x}{r_d\,\mathrm{ctg}\,\alpha+L_\text{к}}\right)^3\right];$$

$$\tau(L)=\frac{r_d\,\mathrm{ctg}\,\alpha}{3v_d}\left[(1+L_\text{к}\,\mathrm{tg}\,\alpha/r_d)^3-1\right]+\frac{L-L_\text{к}}{v_d};$$

$v_d = v_*$ is the velocity of the liquid in the cylindrical part of radius r_d of the collector electrode.

In Figs. 3 and 4 we compare the results of calculations of Δp from (2.3) (solid lines) with the experimental results (dash-and-dot lines) at a supply-voltage pulsation frequency f = 100 Hz for a silicon liquid with $\rho = 850$ kg/m³, $\varepsilon = 2.4$. In the calculation we took $E_0 = 10^7$ V/m, the initial field strength at which discharge is initiated in the liquid, which was estimated from the results of experimental; determination of the initial voltage U_{in} for the given liquid, the results of analog simulation of the electrostatic field of the needle–cone system of electrodes by the electrolytic tank method, also including the results of [8] on the simulation of such an electrostatic field; $r_0 = 0.1$ mm is the radius of the emitter electrode; $r_d = 0.75$ mm is the radius of the receiving aperture of the collector; and $\alpha = 45°$ is the angle between the generatrix of the cone and its height L_K.

The velocity v_d in the receiving aperture of the collector was assumed to be equal to the corresponding flow rate of the working medium. The theoretical and experimental curves were plotted for constant values of the interaction parameter N, determined from $N = L_K\overline{\beta}/v_d$ and characterizing the number of interactions of charges with neutral molecules of the liquid.

The value of the electric relaxation frequency $\overline{\beta}$ was estimated from

$$\overline{\beta}=\frac{I_0}{\varepsilon\varepsilon_0 Uk}\int\limits_0^{L_\text{к}}\frac{dx}{S(x)},$$

where $k = 0.68 + \sqrt{\cot\alpha}$ is the empirical gain of the electric field, obtained from the results of analog simulation of the needle–cone electrode system by the electrolytic tank method.

Comparison of the results of calculation from (2.3) with the experimental data shows that the calculated values of the pressure differ by less than 10% from the experimental values in the region of the optimum length of the collector cone.

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